

Generalized Chernoff Fusion Approximation for Practical Distributed Data Fusion

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Abstract – *This paper advances research in practical distributed data fusion with an emphasis on the generalized fusion of probability density functions in the presence of unknown correlations. Specifically, the proposed algorithm addresses fusion of any finite number of probability density functions in a distributed tracking environment where “rumor propagation” and statistical correlations may be present. This “rumor propagation” arises in real-world tactical military applications where distributed fusion nodes have dynamic and multi-cyclic data flows. In addition, interoperability requirements with legacy systems preclude control over pre-processing of data fusion inputs to ensure statistical independence or modify legacy systems with pedigree tagging techniques. Leveraging the well-known Covariance Intersection algorithm, its generalization, and previously developed approximations to Covariance Intersection, a computationally simple approximation for the generalized fusion of any number of probability density functions is presented as the novel result of this paper. The derivation of this algorithm and numerical examples illustrate that the proposed approach enables practical fusion of generalized (non-Gaussian) observations in an ad-hoc distributed fusion network without the need for pedigree tagging.*

Keywords: Distributed Data Fusion, Tracking, Covariance Intersection, Information Theory, Chernoff Fusion.

1 Introduction

When designing real-world distributed tracking systems for tactical military applications, several practical considerations arise that preclude the use of classical tracking algorithms which assume statistical independence of data inputs. First, tactical environments often consist of sensor and data processing nodes that are connected through mobile ad-hoc networks which are dynamic and unpredictable. Since many of these processing nodes produce fused solutions instead of sensor-level measurements, it is virtually impossible to eliminate redundant data flows between nodes in real-time. Second, many legacy systems that provide tracking data cannot be modified to produce statistically independent updates or provide pedigree information that may assist in identifying

redundant data flows. Finally, tracking data with a variety of statistics such as active, passive, Gaussian, and non-Gaussian are shared between these processing nodes. These factors necessitate generalized algorithms for fusing multiple tracking data inputs of various types in the presence of “rumor propagation”. Without such algorithms, scalable distributed fusion is not possible in tactical military applications.

One approach to handling rumor propagation is called pedigree tagging [1]. This approach involves the exchange of metadata (pedigree) that represents the processing history and source information of a particular piece of tracking data. For example, a track may contain pedigree information that indicates all the sensor sources that contributed to the track as well as the nodes that processed the track. Theoretically, using this approach, algorithms can be developed to identify redundant data at runtime and apply alternative processing to accommodate this redundancy. There are several problems with this approach in practice. First, implementation of a pedigree tagging scheme would require modification of sensor processing systems that produce tracking data. Second, even if redundant data is identified, the pedigree metadata is insufficient to accurately remove redundancies from the track state estimate. Finally, pedigree tagging does not scale with respect to communications bandwidth [2].

An alternative approach to pedigree tagging is Covariance Intersection [3,4,5,6], which was initially developed in the late 1990’s. Covariance Intersection was developed to fuse Gaussian state estimates that may contain redundant (statistically correlated) data. In addition, Covariance Intersection does not require specific knowledge of the statistical correlation between inputs. A generalization of the Covariance Intersection algorithm (Generalized Chernoff Fusion) was later developed for fusion of two arbitrary probability density functions, thus lifting the Gaussian restriction [7]. These ground-breaking developments have enabled scalable distributed data fusion for several important special cases. While additional work has addressed fusion of any number of Gaussian inputs [8], the more generalized case of fusing any number of

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probability density functions has not been previously addressed. This is the emphasis of the current paper.

This paper is organized as follows. Section 2 provides an overview of Covariance Intersection for the Gaussian special case. Section 3 summarizes previous work to extend Covariance Intersection for the fusion of any number of Gaussian inputs. Section 4 provides an overview of Generalized Chernoff Fusion, which has been applied to the case of fusing two probability density functions. Section 5 develops the proposed algorithm that extends previous work to the fusion of any number of probability density functions and provides several approximation techniques for practical implementation. Section 6 summarizes the results and discusses plans for future work.

2 Review of Covariance Intersection

Consider the special case of fusing *two statistically independent state estimates whose probability distributions are Gaussian* and represented by a first and second order moment (mean and covariance matrix). In this case, the fusion equations are given by the information form of the Kalman filter [9,10]. Specifically, the fusion of two statistically independent state estimates with means a and b and covariance matrices A and B results in the fused mean c_{IF} and covariance C_{IF} as follows:

$$\begin{aligned} C_{IF}^{-1} &= A^{-1} + B^{-1} \\ c_{IF} &= C_{IF} (A^{-1}a + B^{-1}b) \end{aligned} \quad (1)$$

Equation (1) is easy to implement and has been used in real-time tracking systems for several decades. Because of its simplicity, it is often used incorrectly in cases where the inputs are not necessarily statistically independent. This incorrect application of Equation (1) leads to fusion results whose covariance is “over optimistic”, which is the most troublesome (yet typical) side effect of rumor propagation in any distributed data fusion architecture.

Historically, the ill effects of rumor propagation have been mitigated by attempting to specify static data flows within the fusion architecture that ensures no rumor propagation. This approach is tenable when dealing with fusion applications where the sensors and processing nodes are under the control of the design agent, such is generally the case for fusion on a single platform. This approach is very difficult to implement as multiple platforms and external tracking systems are integrated into the fusion architecture, primarily because these “external” systems are outside the control of any one design agent. Ironically, as the need and desire for integrating more external systems increases, the problem of rumor propagation becomes unmanageable.

To address this growing problem and enable scalable distributed data fusion, the Covariance Intersection algorithm was developed in the late 1990’s [3,4,5,6]. This algorithm *extends the special case of Gaussian fusion to inputs that have unknown statistical correlation*, as is the case with rumor propagation. The Covariance Intersection equations are a surprisingly simple variant of the classical information filter:

$$\begin{aligned} C_{CI}^{-1} &= \omega A^{-1} + (1 - \omega) B^{-1} \\ c_{CI} &= C_{CI} (\omega A^{-1}a + (1 - \omega) B^{-1}b) \end{aligned} \quad (2)$$

Equation (2) provides a set of solutions, which are a function of the optimization parameter ω on the interval $[0, 1]$. Typically, the solution to Equation (2) is determined by selecting the value of ω that minimizes the determinant of the fused covariance C_{CI} , which has an Information Theoretic justification [7]. An important property of Covariance Intersection is that *the fused solution $\{c_{CI}, C_{CI}\}$ is guaranteed to be consistent for any value of ω , assuming that the inputs $\{a, A\}$ and $\{b, B\}$ are consistent*. Thus, the selection of ω need not be precise, but should attempt to provide a fused covariance C_{CI} that is smaller than that of either input.

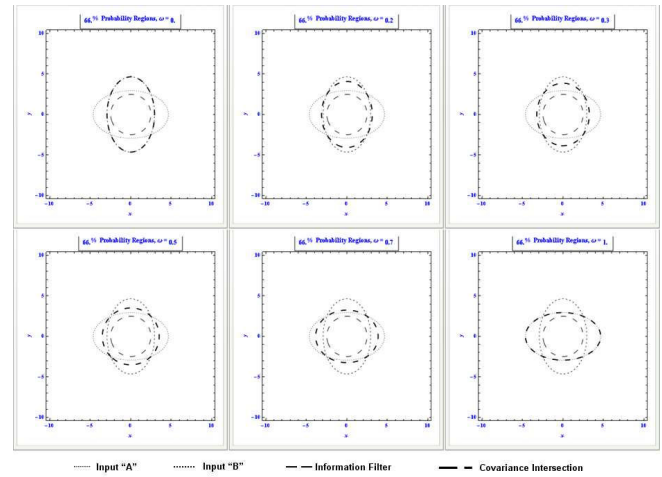


Figure 1. Covariance Intersection solutions for $\omega = \{0.0, 0.2, 0.3, 0.5, 0.7, 1.0\}$ compared to the Information Filter.

Figure 1 provides a simple example of Covariance Intersection solutions as a function of the optimization parameter ω for two Gaussian inputs whose covariance matrices have equivalent determinants. The Information Filter solution (the dashed circle in the center of each plot) given by Equation (1) illustrates the over-optimistic fusion results that may occur if the inputs are incorrectly assumed to be statistically independent. The thick dashed line depicts the result of Equation (2) for each of the specified values

of ω . This illustration demonstrates that the solution for $\omega=0$ and $\omega=1$ reduce to the input B and input A, respectively. In this particular example, the value of ω that minimizes the determinant of the fused covariance is 0.5, which is shown in the bottom left corner of Figure 1.

It is important to note that Equation (2) results in a very simple one-dimensional convex optimization problem that lends itself to real-time implementations. Thus, Covariance Intersection provides a tractable approach to mitigating rumor propagation in the important special case of fusing two Gaussian state estimates.

3 Fast Covariance Intersection

Some applications require the *fusion of $n > 2$ statistically correlated Gaussian state estimates* where each estimate is expressed by a mean and covariance $\{\mu_i, \Sigma_i\}$. While this can be performed by applying Equation (2) iteratively $n-1$ times, it has been noted that this approach can yield different (and less desirable) results than a batch solution given by [8]:

$$\begin{aligned} C^{-1} &= \sum_{i=1}^n \omega_i \Sigma_i^{-1} \\ c &= C \sum_{i=1}^n \omega_i \Sigma_i^{-1} \mu_i \\ \sum_{i=1}^n \omega_i &= 1 \end{aligned} \quad (3)$$

Equation (3) results in an optimization problem for the n values ω_i , where each is restricted to the interval $[0, 1]$, which can be significantly more complex than the optimization problem of Equation (2). In particular, Fränken and Hüpper [8] demonstrate that this optimization becomes difficult when the inputs covariance matrices have radically different eigenvalues. As a result, the following fast approximation was developed in lieu of numerical optimization:

$$\begin{aligned} \omega_i &= \frac{\det[\tilde{P}^{-1}] + \det[\Sigma_i^{-1}] - \det[\tilde{P}^{-1} - \Sigma_i^{-1}]}{n \det[\tilde{P}^{-1}] + \sum_{j=1}^n [\det[\Sigma_j^{-1}] - \det[\tilde{P}^{-1} - \Sigma_j^{-1}]]} \\ \tilde{P}^{-1} &= \sum_{i=1}^n \Sigma_i^{-1} \end{aligned} \quad (4)$$

The terms in Equation (4) have interesting Information Theoretic interpretations that provide insight into the formulation of the optimization parameters ω_i . First,

\tilde{P}^{-1} is the information matrix obtained by fusing all n inputs assuming statistical independence. Secondly, Σ_i^{-1} is the information matrix for the i^{th} state estimate input.

Finally, the term $\tilde{P}^{-1} - \Sigma_i^{-1}$ is the information matrix obtained by fusing all inputs except for the i^{th} one. Thus, the optimization parameters are dictated by the *mutual information* between the information filter solution and each of the inputs. Using the fact that the determinant of the covariance represents the information content of the state estimate, one can also interpret the optimization parameters as the relative information content that each input provides with respect to the total information content obtained by fusing all inputs.

It is important to note that Equation (4) enables simple, real-time implementation of the more general Covariance Intersection problem in terms of the classical Information Filter result. Thus, existing fusion algorithms can be easily “upgraded” to implement Covariance Intersection for n inputs that may suffer from rumor propagation.

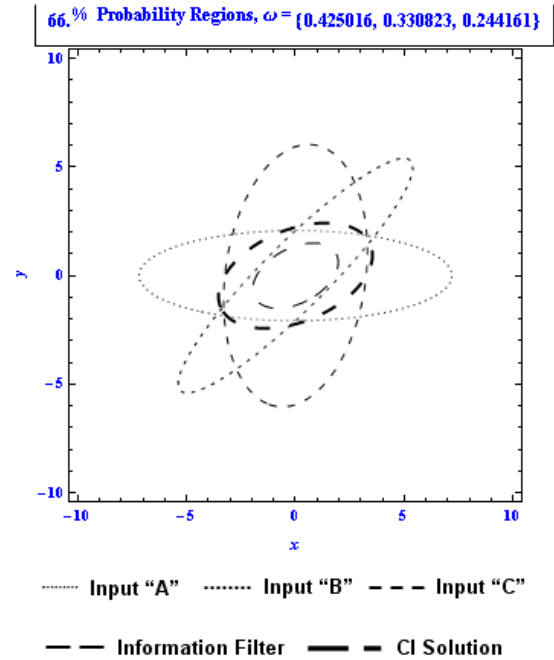


Figure 2. Fast Covariance Intersection solution for three inputs with $\omega_1 \cong 0.425$, $\omega_2 \cong 0.331$, $\omega_3 \cong 0.244$ compared to the Information Filter solution.

Figure 2 illustrates the Covariance Intersection fusion (thick dashed line) of three statistically correlated inputs using Equation (3) and the Information Filter solution (center of the plot) computed using Equation (1). The optimization parameters for each input are computed using Equation (4). As with the previous example in Figure 1, the Covariance Intersection solution has the same shape as the Information

Filter solution, but the Information Filter results in an over optimistic size for the fused covariance.

4 Chernoff Fusion

In the previous sections, the original Covariance Intersection and its generalization to n inputs were discussed. However, in both cases, these algorithms were limited to the case of Gaussian inputs specified by a mean and covariance matrix. To develop a generalized fusion algorithm for any probability density function, the well-known Bayesian fusion serves as a foundation:

$$p_{Bayes}(\bar{x}) = \frac{p_A(\bar{x}) p_B(\bar{x})}{\int p_A(\bar{x}) p_B(\bar{x}) d\bar{x}} \quad (5)$$

Equation (5) provides the Bayesian fusion equation for the *fusion of two arbitrary probability density functions that are assumed to be statistically independent*. In the Gaussian case, Equation (5) takes on a log-linear form equivalent to Equation (1). Thus, the generalized Covariance Intersection (Chernoff Fusion) for fusing two arbitrary probability density functions that have unknown correlation is [7]:

$$p_{GCF}(\bar{x}) = \frac{p_A^\omega(\bar{x}) p_B^{1-\omega}(\bar{x})}{\int p_A^\omega(\bar{x}) p_B^{1-\omega}(\bar{x}) d\bar{x}} \quad (6)$$

Just as with Covariance Intersection, Equation (6) provides a set of solutions, one for each value of the parameter ω . Hurley [7] proposes two criteria for computing the parameter ω : (1) minimize the Shannon Entropy of the fused probability density function and (2) minimize the Chernoff information of the fused probability density function. Hurley demonstrates that minimizing the Shannon Entropy is equivalent to minimizing the determinant of the covariance for the Gaussian case. The Chernoff Information criterion attempts to find the fused probability density function that is in the “middle” of the input probability density functions. While both of these criteria have satisfying Information Theoretic interpretations, there are several practical implementation issues. First, while the Shannon Entropy criterion can be easily extended to the case of more than two inputs, the Chernoff Information extension is not obvious. Secondly, if the Shannon Entropy criterion is used for more than two inputs, the computational complexity is dependent upon the nature of the probability density functions. Generally speaking, this amounts to a multi-dimensional optimization problem that will often contain many local minima. Thus, it may be difficult to achieve computationally feasible fusion in many situations.

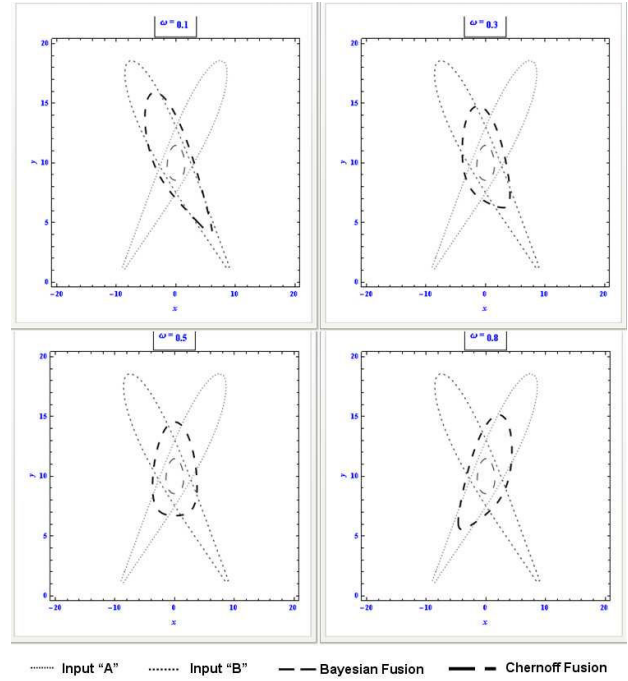


Figure 3. Chernoff Fusion solutions for $\omega = \{.1, .3, .5, .8\}$

Figure 3 illustrates Chernoff Fusion solutions (thick dashed) with respect to ω for two probability density functions (Appendix A) having Shannon Entropies that are identical. The Bayesian Fusion solution (center of each plot) is given by Equation (5). In this example, the value of ω that minimizes both of Hurley's criteria is 0.5.

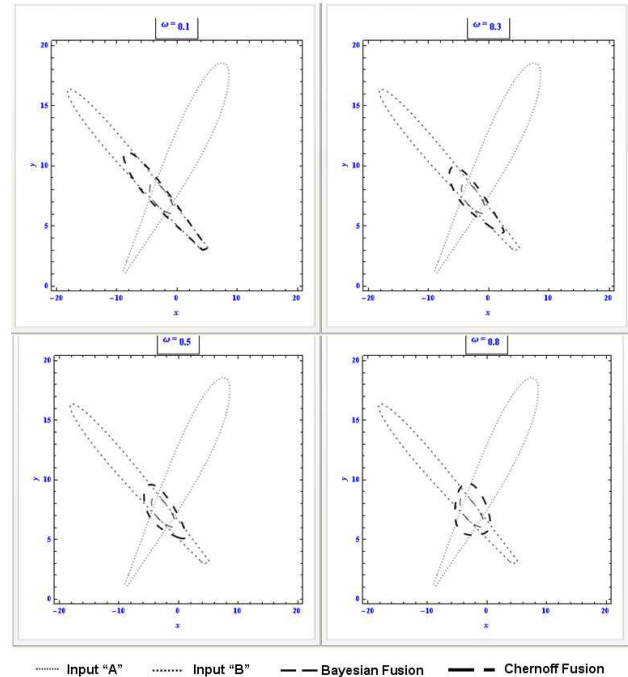


Figure 4. Chernoff Fusion solutions for $\omega = \{.1, .3, .5, .8\}$

Figure 4 depicts another example of Chernoff Fusion for a case where the Shannon Entropies of the inputs are not equal to each other. In this case, the value of ω that minimizes both of Hurley's criteria is 0.47.

5 Generalized Chernoff Fusion

Ultimately, it is desirable to develop an algorithm that handles the most general fusion problem, *fusion of n statistically correlated probability density functions*. In addition, while a theoretical result may be satisfying, a computationally simple solution is necessary to achieve practical, scalable distributed fusion. Starting with Equation (6), the obvious extension to Chernoff Fusion for multiple inputs is given by:

$$p_{GCF}(\bar{x}) = \frac{\prod_{i=1}^n p_i^{\omega_i}(\bar{x})}{\int \prod_{i=1}^n p_i^{\omega_i}(\bar{x}) d\bar{x}} \quad (7)$$

$$\sum_{i=1}^n \omega_i = 1$$

where the optimization parameters ω_i need to be computed using some criteria. As noted in the previous section, the implementation of this optimization problem may be too complex for practical implementations.

After reviewing previous research results outlined in Sections 3-4, there are several notable observations that indicate an approximate solution to the Generalized Chernoff Fusion problem exists:

- The optimization parameters in Equation (4) *depend upon the determinant of the covariance matrix* for a Gaussian distribution.
- The *determinant of the covariance matrix is related to the Shannon Entropy* for a Gaussian distribution.
- The “optimal” value of ω in Equation (6) *depends upon the Shannon Entropy of the inputs*.

This analysis seems to indicate that an analogous formula to Equation (4) exists and that it is a function of the Shannon Entropy of the arbitrary probability density function inputs. With this goal in mind, the following are noted. First, the Shannon Entropy H for an m -variate Gaussian distribution is given in terms of its covariance Σ by:

$$H_{Gaussian} = \ln \left[\sqrt{(2\pi e)^m \det[\Sigma]} \right] \quad (8)$$

Secondly, the determinant has the following property:

$$\det[\Sigma^{-1}] = \frac{1}{\det[\Sigma]} \quad (9)$$

Putting Equation (8) and (9) together, we obtain the following useful relationship between the covariance matrix of a Gaussian probability density function and its Shannon Entropy:

$$\det[\Sigma^{-1}] = (2\pi)^{m/2} e^{(m/2) - H_{Gaussian}} \quad (10)$$

Furthermore, following the same Information Theoretic interpretation provided after Equation (4), we define the Shannon Entropies: H_B for the Bayesian fusion of all n , H_i for the i^{th} input, and H_{B-i} for the Bayesian fusion of all except the i^{th} input. Inserting Equation (10) into Equation (4) and simplifying, we obtain (Appendix B):

$$\omega_i = \frac{1 - e^{H_B - H_{B-i}} + e^{H_B - H_i}}{n + \sum_{j=1}^n [e^{H_B - H_j} - e^{H_B - H_{B-i}}]} \quad (11)$$

Just as with Equation (4), Equation (11) computes the optimization parameters using the relative information content of each of the inputs compared to the fused result. In particular, the term $H_{B-i} - H_B$ is the information increase due to the inclusion of the i^{th} input while the term $H_i - H_B$ is the information increase due to the inclusion of all data except for the i^{th} input.

Although Equation (11) is far simpler than performing multi-parameter optimization, it still may not provide a practical means of computing the optimization parameters ω_i . To implement Equation (11) as written, the following steps are required:

1. Compute the Shannon Entropy H_i of each of the n input probability density functions.
2. Compute $n+1$ Bayesian Fusion solutions: one solution that contains all n inputs; and n other solutions, each containing all but the i^{th} input.
3. Compute the Shannon Entropies of each of the Bayesian fusion solutions described in the previous step and evaluate Equation (11).

With these steps, Equation (11) can be evaluated using an existing Bayesian Fusion algorithm and computing Shannon Entropies.

As an alternative to the approach outlined above, a few very simple approximations can be made to expedite computations. First, we may assume (as a lower limit) that the Shannon Entropy of the Bayesian Fusion results is equal to the input with the smallest Entropy divided by the number of inputs “ n ”. That is, we can make the following lower limit approximations:

$$H_B \approx \frac{\min[H_i]}{n} \quad (12)$$

$$H_{B-i} \approx \frac{\min[H_{j,i \neq j}]}{n-1}$$

Using Equation (12), we can reduce Equation (11) to:

$$\omega_i = \frac{1 - e^{-\frac{\min[H_i]}{n}} + e^{-\frac{\min[H_i]}{n} H_i}}{n + \sum_{j=1}^n \left[e^{-\frac{\min[H_i]}{n} H_i} - e^{-\frac{\min[H_i]}{n} \frac{\min[H_{j,i \neq j}]}{n-1}} \right]} \quad (13)$$

Equation (13) provides the optimization parameters in terms of only the Shannon Entropies of the probability density function inputs. Thus, it provides a tractable solution for Generalized Chernoff Fusion.

To validate Equation (13), several fusion examples were developed and the results were compared with the “optimal” solution obtained using numerical optimization techniques.

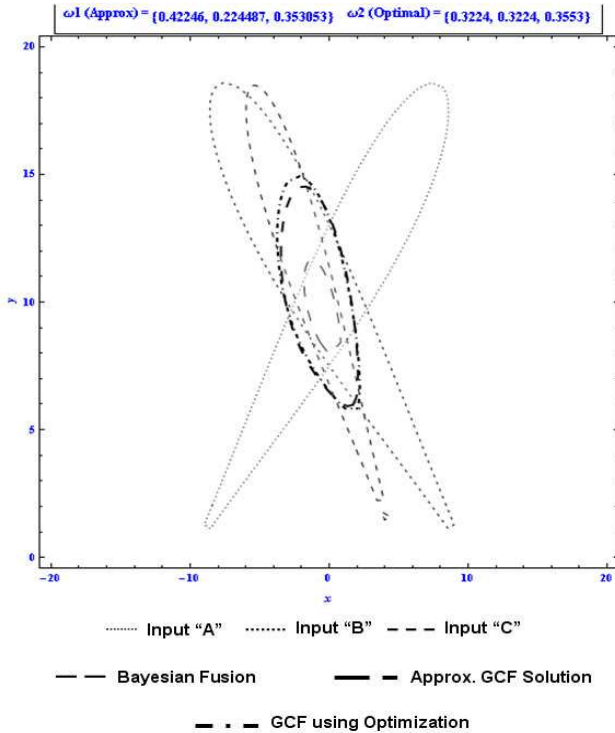


Figure 5. Generalized Chernoff Fusion for three PDFs

Figure 5 illustrates the Generalized Chernoff Fusion of three probability density functions. The approximate solution obtained using Equation (13) is shown with the thick dashed line while the numerical optimization result is shown with the thick dot-dashed line. For comparison, the Bayesian Fusion result for all three inputs is also depicted. Equation (13) resulted in the following optimization parameters:

$$\omega_A(\text{Approx}) \cong 0.42 \quad \omega_B(\text{Approx}) \cong 0.23 \quad (14)$$

$$\omega_C(\text{Approx}) \cong 0.35$$

while the numerical optimization resulted in:

$$\omega_A(\text{Optimal}) \cong 0.32 \quad \omega_B(\text{Optimal}) \cong 0.32 \quad (15)$$

$$\omega_C(\text{Optimal}) \cong 0.36$$

It is evident from Figure 5 that the solutions are not very different from each other, although the values of the solutions do differ. However, given the extremely simple Equation (13) as compared to the general problem of numerical optimization, these minor discrepancies seem acceptable in order to achieve a fast approximate solution.

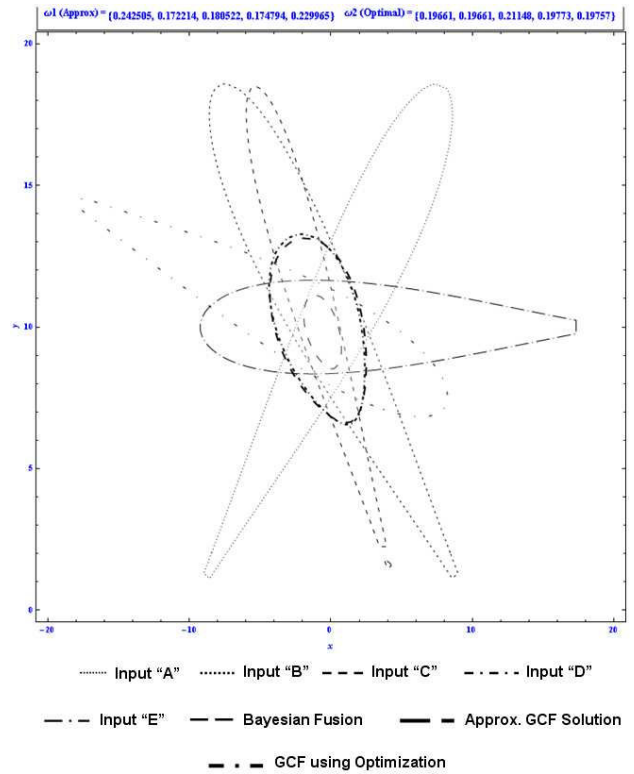


Figure 6. Generalized Chernoff Fusion for five PDFs

Figure 6 provides an additional example of the Generalized Chernoff Fusion Approximation (thick dashed line), the Generalized Chernoff Fusion due to numerical optimization (thick dot-dashed line) as well as the Bayesian Fusion solution (egg-shaped dashed line in the center of the plot).

In this case, the optimization parameters for the five input probability density functions from Equation (13) are:

$$\begin{aligned}\omega_A(\text{Approx}) &\cong 0.24 & \omega_B(\text{Approx}) &\cong 0.17 \\ \omega_C(\text{Approx}) &\cong 0.18 & \omega_D(\text{Approx}) &\cong 0.18 \\ \omega_E(\text{Approx}) &\cong 0.23\end{aligned}\quad (16)$$

while the numerical optimization resulted in:

$$\begin{aligned}\omega_A(\text{Optimal}) &\cong 0.20 & \omega_B(\text{Optimal}) &\cong 0.20 \\ \omega_C(\text{Optimal}) &\cong 0.21 & \omega_D(\text{Optimal}) &\cong 0.20 \\ \omega_E(\text{Optimal}) &\cong 0.19\end{aligned}\quad (17)$$

Again, the solutions are quite similar while the computational complexity is vastly different.

6 Summary and Future Work

Leveraging previous work beginning with the original Covariance Intersection algorithm and subsequent extensions to handle multiple Gaussian inputs and pairs of non-Gaussian probability density functions, a fast approximation for the general case was developed. This approximation was derived by noting: (1) the relationship between the Shannon Entropy and the determinant of the Gaussian covariance and (2) that the fast approximation previously developed (Equation 4) captured the relative information content of each input with respect to the Bayesian fused solution. The novel contribution of this paper was compared to numerical optimization for validation and it was found that the Generalized Chernoff Fusion solutions produced very similar probability regions. Future work will investigate the degree to which these solutions differ in order to further validate the fast approximation. In addition, this approximation will be verified using extreme limiting situations such as cases where the Shannon Entropies of the input probability density functions are both very large and very small.

7 Appendix A

The Probability Density Functions (PDFs) presented in Figures (3)–(6) are given by a product of PDFs defined in Polar coordinates (range “ r ” and bearing “ b ”) converted into Cartesian coordinates (x and y) using a simple transformation of variables from Polar to Cartesian. The Polar coordinate representations of these PDFs are given by:

$$p_r(r) = \frac{1}{(r_{\min} + r_{\max})\pi} (\arctan[-(r - r_{\max})] + \arctan[r - r_{\min}])$$

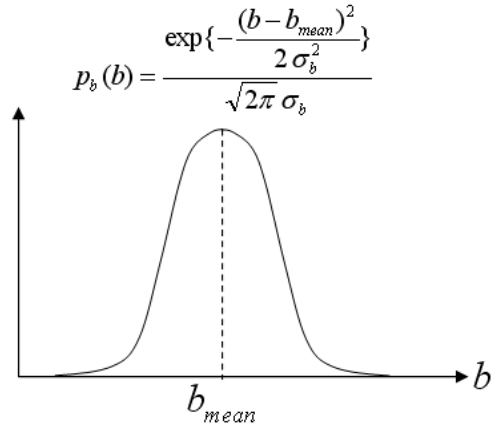
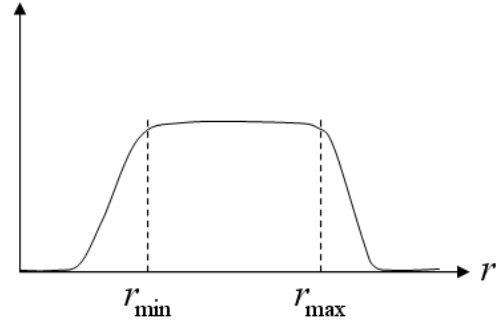


Figure 7. PDFs used in Figures 3-6 in Polar Coordinates

Using these PDFs, the Cartesian PDFs are given by:

$$p(x, y) = p_r(r(x, y)) \bullet p_b(b(x, y))$$

where

$$r(x, y) = \sqrt{x^2 + y^2}$$

$$b(x, y) = \arctan\left[\frac{y}{x}\right]$$

Figures (3)–(6) provide several examples where the parameters “ r_{\min} ”, “ r_{\max} ”, “ b_{mean} ”, and “ σ_b ” are varied to produce probability density functions reminiscent of observation likelihoods for passive acoustic or electronic warfare sensors.

8 Appendix B

Starting with Equation (4) and using the relations defined in Equations (8) and (10), we can express Equation (4) in terms of the Shannon Entropies H_B , H_i , and H_{B-i} :

$$\begin{aligned}\omega_i &= \frac{\det[\tilde{P}^{-1}] + \det[\Sigma_i^{-1}] - \det[\tilde{P}^{-1} - \Sigma_i^{-1}]}{n \det[\tilde{P}^{-1}] + \sum_{j=1}^n [\det[\Sigma_j^{-1}] - \det[\tilde{P}^{-1} - \Sigma_j^{-1}]]} \\ &= \frac{(2\pi)^{m/2} e^{(m/2)} (e^{-H_B} + e^{-H_i} - e^{-H_{B-i}})}{(2\pi)^{m/2} e^{(m/2)} \left(n e^{-H_B} + \sum_{j=1}^n [e^{-H_i} - e^{-H_{B-i}}] \right)}\end{aligned}$$

Canceling common factors in the numerator and denominator, we arrive at Equation 11:

$$\begin{aligned}\omega_i &= \frac{(2\pi)^{m/2} e^{(m/2)} (e^{-H_B} + e^{-H_i} - e^{-H_{B-i}})}{(2\pi)^{m/2} e^{(m/2)} \left(n e^{-H_B} + \sum_{j=1}^n [e^{-H_i} - e^{-H_{B-i}}] \right)} \\ &= \frac{1 - e^{H_B - H_{B-i}} + e^{H_B - H_i}}{n + \sum_{j=1}^n [e^{H_B - H_i} - e^{H_B - H_{B-i}}]}\end{aligned}$$

9 References

- [1] Ceruti, M.G.; Wright, T.L.; Powers, B.J.; McGirr, S.C., "Data Pedigree and Strategies for Dynamic Level-One Sensor Data Fusion," *Information Fusion, 2006 9th International Conference on*, vol., no., pp.1-5, 10-13 July 2006.
- [2] Nicholson, D.; Lloyd, C.M.; Julier, S.J.; Uhlmann, J.K., "Scalable distributed data fusion," *Information Fusion, 2002. Proceedings of the Fifth International Conference on*, vol.1, no., pp. 630-635 vol.1, 2002.
- [3] Jeffrey K. Uhlmann, "General Data Fusion for Estimates with Unknown Cross Covariances." *SPIE vol. 2755*, 1996, pp. 536-547.
- [4] J. K. Uhlmann, J. K. Julier, and M. Csorba, "Nondivergent simultaneous map-building and localization using covariance intersection." *Proc. SPIE -Int. Soc. Opt. Eng.*, vol. 3087, April 1997, pp. 2-11.
- [5] S. J. Julier and J. K. Uhlmann, "A Non-divergent estimation algorithm in the presence of unknown correlations." *Proc. 1997 Am. Control Conf.*, v. 4, June 1997, pp. 2369-2373.
- [6] Jeffrey Uhlmann, Simon Julier, Behzad Kamgar-Parsi, Marco Lanzagorta, and Haw-Jye Shyu, "The NASA Mars Rover: A Testbed for Evaluating Applications of Covariance Intersection." *SPIE Con. Unmanned Ground Vehicle Tech., Orlando, Florida*, vol. 3693, April 1999, pp. 140-149.
- [7] Hurley, M.B., "An information theoretic justification for covariance intersection and its generalization," *Information Fusion, 2002. Proceedings of the Fifth International Conference on*, vol.1, no., pp. 505-511 vol.1, 2002.
- [8] Franken, D.; Hupper, A., "Improved fast covariance intersection for distributed data fusion," *Information Fusion, 2005. 8th International Conference on*, vol.1, no., pp. 7 pp.-, 25-28 July 2005.
- [9] J. Manyika and H.F. Durrant-Whyte, *Data Fusion and Sensor Management: An Information-Theoretic Approach*, Prentice Hall, 1994.
- [10] S. Grime and H.F. Durrant-Whyte, "Data fusion in decentralized networks," *Control Eng. Practice*, vol. 2: pp. 849-863, 1994.